5. KONOPLEV V.A. Matrix methods in the kinematics of the complex motion of a rigid body and a system of rigid bodies. Sbornik nauchno-metoicheskikh statei po teoreticheskoi mekhanike. Moscow, Vyssh. Shk., 17, 1986.
6. LUR'YE A.I., Analytical Mechanics. Moscow, Fizmatgiz, 1961.
7. WITTENBURG J., Dynamics of Systems of Rigid Bodies. Stuttgart, B.G. Teubner, 1977.
8. LITVIN-SEDOI M.Z., Mechanics of systems of connected rigid bodies. Itogi nauki i tekhniki. Ser. Obshchaya mekhanika, Moscow, VINITI, 5, 1982.
9. SARYCHEV V.A., Problems of the Orientation of Artificial Satellites. Itogi nauki i tekhniki. Ser. Issledovaniye kosmicheskogo prostranstva. 11, Moscow, VINITI, 1978.

Translated by L.K.

PMM U.S.S.R.,Vol.51,No.5,pp.600-606,1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

AN ANISOTROPICALLY ELASTIC SPHERE IN FREE MOTION*

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Corrections are found to the inertial tensor components of a rotating sphere in the case of small anisotropy of its elastic properties of general form. Singularities in the behaviour of the freely rotating sphere due to the intrinsic elasticity are discussed in specific examples. Without making any assumptions on the smallness of the anisotropy, the strain is calculated for a sphere having a plane of isotropy. It was shown /1, 2/ in the problem of the motion of a solid deformable body around a centre of mass under the simplifying assumptions that the natural vibration frequencies greatly exceed the angular velocity while the internal friction forces ensure sufficiently rapid damping of the natural vibrations, that taking account of the intrinsic elasticity of the body is equivalent to the action of a moment on it proportional to the fourth power of the angular velocity component and calculated by means of the solution of the quasistatic problem of the deformation of a rotating body.

The moment corresponding to the influence of the intrinsic elasticity has been calculated /3/ for a body close in shape to a sphere. The homogeneous anisotropically elastic sphere in free motion considered in this paper is still another example when the solution of this problem can be obtained by analytic means. The representation of the behaviour of this system can turn out to be useful when considering questions of the earth's motion in connection with the hypothesis that the earth has the features of a complex crystal.

Let us represent the stress tensor in the form

$$\sigma_{ij} = \lambda u_{ll}\delta_{ij} + 2\mu u_{ij} + c_{ijnm}u_{nm}$$

where u_{ij} is the strain tensor, λ and μ are Lamé constants, and u_i are the components of the displacement vector. The tensor of the elastic constants c_{ijnm} satisfies the following symmetry conditions /5/:

$$c_{ijnm} = c_{jinm} = c_{ijmn} = c_{nmij}$$

and has 21 indepdent components in the most general case of an anisotropic linearly elastic body.

We shall consider only the almost Eulerian motions of a deformable body. This is possible if it is sufficiently rigid and the vibrations of the elastic body that occur damp out rapidly /2/. The elastic constants are such that the following inequalities are satisfied

$$\varepsilon \ll \delta \ll 1 \quad (\varepsilon = \rho R^2/(\mu t_*^2)) \tag{1}$$

where ρ is the density, R is the radius of the sphere, t_* is the characteristic time of sphere motion as a whole, and δ is the ratio of the greatest of the elastic constants c_{ijnm} to

*Prikl.Matem.Mekhan.,51,5,767-774,1987

$(c_{ijnm} = \mu\delta a_{ijnm})$.

We represent the displacement vector $\mathbf{u}(\mathbf{r}, t)$ in the form of a series in the small parameters ε and δ and we confine ourselves to just the first terms therein

$$\mathbf{u}(\mathbf{r}, t) = \varepsilon [\mathbf{u}_0(\mathbf{r}, t) + \delta \mathbf{u}_1(\mathbf{r}, t)]$$

Under assumptions (1) the elastic displacements $\mathbf{u}_0(\mathbf{r}, t)$ and $\mathbf{u}_1(\mathbf{r}, t)$ can be evaluated in the $Ox_1x_2x_3$ coordinate system, fixed in the undeformed sphere /3/.

The displacement vector $\mathbf{u}(\mathbf{r}, t)$ is determined as a result of solving the following problems.

1°. The problem of the deformation of a rotating, homogeneous, isotropic sphere

$$\partial\sigma_{ij}^\circ/\partial x_j = f_i \text{ in } V;\ \sigma_{ij}^\circ n_j = 0 \text{ on } S \tag{2}$$

$$\overset{\circ}{\sigma}_{ij} = \varkappa \frac{\partial u_{0l}}{\partial x_l}\delta_{ij} + \frac{\partial u_{0i}}{\partial x_j} + \frac{\partial u_{0j}}{\partial x_i}, \quad \varkappa = \frac{\lambda}{\mu}$$

$$\mathbf{f} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

($\boldsymbol{\omega}$ is the angular velocity, and $\mathbf{n}$ is the vector normal to the surface). Here and henceforth, R, t_*, are taken as scale factors and M is the mass of the sphere

2°. The problem of calculating $\mathbf{u}_1(\mathbf{r}, t)$ for known $\mathbf{u}_0(\mathbf{r}, t)$.

$$\varkappa \frac{\partial u_{ll}^1}{\partial x_j}\delta_{ij} + 2\frac{\partial u_{ij}^1}{\partial x_j} = -a_{ijmn}\frac{\partial \overset{\circ}{u}_{mn}}{\partial x_j} \quad \text{in } V \tag{3}$$

$$(\varkappa u_{ll}^1\delta_{ij} + 2u_{ij}^1)x_j = -a_{ijmn}\overset{\circ}{u}_{mn}x_j \quad \text{on } S$$

$$\overset{\circ}{u}_{ij} = \frac{1}{2}\left(\frac{\partial u_{0i}}{\partial x_j} + \frac{\partial u_{0j}}{\partial x_i}\right), \quad u_{ij}^1 = \frac{1}{2}\left(\frac{\partial u_{1i}}{\partial x_j} + \frac{\partial u_{1j}}{\partial x_i}\right)$$

Taking account of the elastic properties of the body, the angular velocity $\boldsymbol{\omega}$ is determined as a result of solving the equation

$$\mathbf{K}^{\cdot} + \boldsymbol{\omega} \times \mathbf{K} = 0$$

where the components of the kinetic moment vector $\mathbf{K}$ (taking into account only terms linear in $\mathbf{u}$) are determined as follows:

$$K_l = I_{lm}\omega_m$$

$$I_{lm} = \int\limits_V [(x_k^2 + 2x_ku_k)\delta_{lm} - (x_lx_m + 2x_lu_m)]\, dV = \overset{\circ}{I}_{lm} + \varepsilon(I_{lm}' + \delta I_{lm}'')$$

We will use the solution of the problem (2) presented in /3/. We have in a Cartesian system of coordinates

$$\begin{aligned}
u_{01} = {} & x_1\left[\omega^2\left(\frac{1}{12} - \frac{1}{6}\xi_1 + \xi_2 + 2\xi_3\right) + \omega_1^2\frac{1}{2}\left(\xi_1 - \frac{1}{2}\right)\right] - \\
& (x_2\omega_1\omega_2 + x_3\omega_1\omega_3)\left[\frac{1}{4} - \frac{1}{2}\xi_1 + \frac{1}{2}\left(3\xi_1 - \frac{1}{2}\right)\times\right. \\
& \left.(x_2^2 + x_3^2 - x_1^2) - 2\xi_1x_1^2\right] + x_1x_2x_3\omega_2\omega_3\left(5\xi_1 - \frac{1}{2}\right) - \\
& x_1(x_1^2 + x_2^2 + x_3^2)\omega^2\left(\frac{\xi_1}{3} + \xi_2\right) + \\
& \omega_1^2x_1\left[\xi_1x_1^2 + \frac{1}{4}(1 - 6\xi_1)(x_2^2 + x_3^2)\right] + \\
& \frac{1}{4}x_1[x_2^2\omega_2^2 + x_3^2\omega_3^2](10\xi_1 - 1)
\end{aligned}$$

$$\xi_1 = \frac{3\varkappa + 2}{2(19\varkappa + 14)}, \quad \xi_2 = \frac{1}{15(\varkappa + 2)}, \quad \xi_3 = \frac{1}{15(3\varkappa + 2)}$$

Cyclic permutation of x_1, x_2, x_3 and $\omega_1, \omega_2, \omega_3$ yields u_{02} and u_{03}.

The solution of the problem (3) is preceded by the calculation of the expressions on the right-hand sides of the equations and boundary conditions. The expressions are not presented because of their awkwardness.

Utilizing the fact that $\partial u_{mn}^\circ/\partial x_j$ are linear in x_1, x_2, x_3 while $u_{mn}^\circ x_j$ contains linear and cubic terms in the space coordinates, we represent the problem (3) in the following form

$$(\varkappa + 1)\operatorname{grad}\operatorname{div}\mathbf{u}_1 + \Delta\mathbf{u}_1 = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 \text{ in } V \tag{4}$$

$$\left[\varkappa\frac{\partial u_{1l}}{\partial x_l}\delta_{ij} + \frac{\partial u_{1i}}{\partial x_j} + \frac{\partial u_{1j}}{\partial x_l}\right]x_j = b_{ij}x_j + b_{inkl}x_1^nx_2^kx_3^l; \tag{5}$$

$$n + k + l = 3 \quad \text{on } S$$

The quantities a_{ij}, b_{ij}, b_{inkl} in these relationships include 21 elastic constants a_{ijnm} in the general case.

Starting from the equations and boundary conditions (4) and (5), it can be concluded that $\mathbf{u}_1(\mathbf{r}, t)$ should be sought in the form of polynomials

$$\mathbf{u}_1 = \mathbf{A}_n x_n + \mathbf{B}_{kmp} x_1{}^k x_2{}^m x_3{}^p$$
$$n, k, m, p = 1, 2, 3; \quad k + m + p = 3$$

It is necessary to determine 39 quantities A_{in} and B_{ikmp}.

After substituting $\mathbf{u}_1$ into (4), equating terms in x_1, x_2, x_3 to zero, we obtain 9 equatios. The boundary conditions (5) yield 3 relationships that satisfy $x_1^2 + x_2^2 + x_3^2 = 1$ on the surface of the sphere. Each of them can be represented in the form

$$\sum_{i=1}^{3} x_i \left[D_{i0} + x_1^2 D_{i1} + x_2^2 D_{i2} + x_3^2 D_{i3}\right] + x_1 x_2 x_3 D_4 = 0$$

where D_{ij} and D_4 contain unknown expansion coefficients A_{in}, B_{ikmp} and the parameters a_{ijlm}. Requiring that the equation for the surface of the sphere be extracted in the expressions in the square brackets, and that the expression with $x_1 x_2 x_3$ vanish, we obtain 30 algebraic equations.

The system of 39 equations from which the quantities A_{in} and B_{ipkm} are determined are separated into four groups of equations. Coefficients evaluated independently of the others and, respectively, determining the additions to the diagonal components of the inertial tensor I_{ii}'' and to the centrifugal moments of inertia I_{12}'', I_{13}'', I_{23}'' are in each of the groups.

The solution of problem (4), (5) results in the following expressions for the corrections to the inertial tensor components

$$\begin{aligned}
I_{11}'' = 2/35\, \{ & \omega^2 [\varepsilon_1 a_{1111} + \varepsilon_4 (a_{1122} + a_{1133}) + \varepsilon_9 (a_{1212} + \\
& a_{1313}) + \varepsilon_{14} (a_{2222} + a_{3333}) + \varepsilon_{17} a_{2233} + \varepsilon_{19} a_{2323}] + \\
& \omega_1^2 [\varepsilon_2 a_{1111} + \varepsilon_5 (a_{1122} + a_{1133}) + \varepsilon_{10} (a_{1212} + a_{1313}) + \\
& \varepsilon_{15} (a_{2222} + 2a_{2233} + a_{3333})] + \omega_2^2 [\varepsilon_6 a_{1122} + \varepsilon_7 a_{1133} + \\
& \varepsilon_{10} a_{1212} + \varepsilon_{16} a_{2222} + \varepsilon_{18} a_{2233} + (\varepsilon_{15} + {}^1/{}_2\varepsilon_{29})\, a_{3333} + (\varepsilon_{10} + \\
& \varepsilon_{28})\, a_{2323}] + \omega_3^2 [\varepsilon_7 a_{1122} + \varepsilon_6 a_{1133} + \varepsilon_{10} a_{1313} + (\varepsilon_{15} + \\
& {}^1/{}_2\varepsilon_{29})\, a_{2222} + \varepsilon_{18} a_{2233} + \varepsilon_{16} a_{3333} + (\varepsilon_{10} + \varepsilon_{28})\, a_{2323}] + \\
& \omega_1\omega_2 [\varepsilon_3 a_{1112} + \varepsilon_{11} a_{1222} + \varepsilon_{12} a_{1233} + (2\varepsilon_{10} + \varepsilon_{28})\, a_{1323}] + \\
& \omega_1\omega_3 [\varepsilon_3 a_{1113} + (2\varepsilon_{10} + \varepsilon_{28})\, a_{1223} + \varepsilon_{12} a_{1322} + \varepsilon_{11} a_{1333}] + \\
& \omega_2\omega_3 [\varepsilon_8 a_{1123} + 2\varepsilon_{10} a_{1213} + \varepsilon_{13} (a_{2223} + a_{2333})]\}
\end{aligned}$$

$$\begin{aligned}
I_{12}'' = -1/35\, \{ & -\omega^2 [(\varepsilon_{11} + \varepsilon_{21})\, (a_{1112} + a_{1222}) - \varepsilon_{26} a_{1233} + \\
& (2\varepsilon_{10} + \varepsilon_{28})\, a_{1323}] + \omega_1^2 [(\varepsilon_{21} + {}^1/{}_2\varepsilon_{25})\, a_{1112} + \varepsilon_{21} a_{1222} + \\
& (\varepsilon_{21} - \varepsilon_{28})\, a_{1233}] + \omega_2^2 [\varepsilon_{21} a_{1112} + (\varepsilon_{21} + {}^1/{}_2\varepsilon_{25})\, a_{1222} + (\varepsilon_{21} - \\
& \varepsilon_{28})\, a_{1233}] + \omega_3^2 [\varepsilon_{22} (a_{1112} + a_{1222}) + \varepsilon_{27} a_{1233} + 2\varepsilon_{26} a_{1323}] + \\
& \omega_1\omega_2 [\varepsilon_{20} (a_{1111} + 2a_{1122} + a_{2222}) + \varepsilon_{24} (a_{1133} + a_{2233}) + \\
& \varepsilon_{25} a_{1212} + \varepsilon_{28} (a_{1313} + a_{2323}) + \varepsilon_{29} a_{3333}] + \omega_1\omega_3 [\varepsilon_{23} a_{1123} + \\
& (\varepsilon_{25} - \varepsilon_{28})\, a_{1213} + (\varepsilon_{23} - \varepsilon_{28})\, (a_{2223} + a_{2333})] + \\
& \omega_2\omega_3 [\varepsilon_{23} a_{1322} + (\varepsilon_{23} - \varepsilon_{28})\, (a_{1113} + a_{1333}) + (\varepsilon_{25} - \\
& \varepsilon_{28})\, a_{1223}]\}
\end{aligned}$$

$$\varepsilon_1 = \frac{11}{72} - \frac{3}{2}\xi_1 + \frac{7}{3}\xi_3 + \frac{61}{6}\xi_1^2 + 4\xi_1\xi_2 - 12\xi_2^2 - 140\xi_3^2$$

$$\varepsilon_2 = -\frac{11}{24} + \frac{9}{2}\xi_1 + 7\xi_3 - \frac{61}{2}\xi_1^2 + 12\xi_1\xi_2$$

$$\varepsilon_3 = -\frac{2}{3} + 2\xi_1 + 42\xi_3 - 12\xi_1^2 + 24\xi_1\xi_2$$

$$\varepsilon_4 = \frac{91}{36} - \frac{91}{4}\xi_1 + 49\xi_3 + \frac{4}{9}\xi_1^2 + \frac{2''}{3}\xi_1\xi_2 - 56\xi_2^2 - 280\xi_3^2$$

$$\varepsilon_5 = -\frac{43}{24} + \frac{107}{4}\xi_1 + 14\xi_3 - \frac{86}{3}\xi_1^2 + 4\xi_1\xi_2$$

$$\varepsilon_6 = -\frac{49}{24} + \frac{83}{4}\xi_1 + 14\xi_3 + \frac{52}{3}\xi_1^2 + 4\xi_1\xi_2$$

$$\varepsilon_7 = -\frac{41}{24} + \frac{83}{4}\xi_1 + 7\xi_3 + 10\xi_1^2$$

$$\varepsilon_8 = -\frac{2}{3} + 14\xi_3 + \frac{44}{3}\xi_1^2 + 8\xi_1\xi_2$$

$$\varepsilon_9 = 8\left(\frac{1}{9}\xi_1^2 - \frac{1}{3}\xi_1\xi_2 - 2\xi_2^2\right)$$
$$\varepsilon_{10} = 4\xi_1(2\xi_2 - {}^1/_3\xi_1)$$
$$\varepsilon_{11} = {}^1/_3 + 14\xi_1 - 6\xi_1^2 + 24\xi_1\xi_2$$
$$\varepsilon_{12} = {}^1/_3 + 28\xi_1 - 14\xi_3 - 22/3\xi_1^2 + 8\xi_1\xi_2$$
$$\varepsilon_{13} = {}^1/_3 - 2\xi_1 + 14\xi_3 + 18\xi_1^2 + 24\xi_1\xi_2$$
$$\varepsilon_{14} = \frac{35}{6} - \frac{455}{18}\xi_1 - \frac{49}{6}\xi_3 - \frac{4}{3}\xi_1^2 - 8\xi_1\xi_2 - 12\xi_2^2$$
$$\varepsilon_{15} = -\frac{11}{12} + \frac{25}{2}\xi_1 + \frac{7}{2}\xi_3 - 10\xi_1^2$$
$$\varepsilon_{16} = -\frac{5}{8} + 9\xi_1 + \frac{21}{2}\xi_3 + \frac{23}{2}\xi_1^2 + 12\xi_1\xi_2$$
$$\varepsilon_{17} = \frac{77}{36} - \frac{511}{18}\xi_1 - \frac{49}{3}\xi_3 - \frac{8}{9}\xi_1^2 - 8\xi_2^2 - 560\xi_3^2 - \frac{16}{3}\xi_1\xi_2 - 140\xi_2\xi_3$$
$$\varepsilon_{18} = -\frac{17}{12} + 19\xi_1 + 14\xi_3 + \frac{34}{3}\xi_1^2 + 4\xi_1\xi_2$$
$$\varepsilon_{19} = -\frac{16}{9}(\xi_1 + 3\xi_2)^2, \quad \varepsilon_{20} = \frac{1}{4}(1 - 14\xi_1)^2$$
$$\varepsilon_{21} = \frac{1}{4} - \frac{7}{2}\xi_1 + 14\xi_1^2, \quad \varepsilon_{22} = \frac{1}{4} - \frac{7}{6}\xi_1 - 18\xi_1^2$$
$$\varepsilon_{23} = 2\xi_1(14\xi_1 - 1), \quad \varepsilon_{24} = \frac{1}{2} - 12\xi_1 + 70\xi_1^2$$
$$\varepsilon_{25} = 2(1 - 2\xi_1 + 6\xi_1^2), \quad \varepsilon_{26} = -\frac{7}{12} + \frac{7}{2}\xi_1 - 14\xi_3 - \frac{8}{3}\xi_1^2 - 8\xi_1\xi_2, \quad \varepsilon_{27} = -\frac{9}{4} - \frac{7}{2}\xi_1 + 142\xi_1^2$$
$$\varepsilon_{28} = 4\xi_1^2, \quad \varepsilon_{29} = {}^1/_4 - 5\xi_1 + 25\xi_1^2$$

The expressions for the remaining corrections to the inertial tensor components are obtained as a result of cyclic permutation of $\omega_1, \omega_2, \omega_3$ and the subscripts $1, 1, 2, 3$ in a_{ijkl} (the symmetry properties are used here).

The inertia tensor of an anisotropic elastic sphere has the form

$$I_{ij} = I_{ij}^\circ + \varepsilon(I_{ij}' + \delta I_{ij}''); \quad I_{ij}^\circ = = \frac{2}{5}\delta_{ij}$$
$$I_{ij}' = \frac{2}{35}\left[\omega^2\left(-\frac{1}{3} + \frac{\xi_1}{3} + 4\xi_2 + 28\xi_3\right)\delta_{ij} + (1 - \xi_1)\omega_i\omega_j\right]$$

where I_{ij}° is the inertial tensor of a non-deformable sphere, and I_{ij}' is the correction to the inertial tensor related to the deformation $\mathbf{u}_0$.

The quantities I_{ij}', I_{ij}'' are quadratic functions of the angular velocity components. The components of the kinetic moment vector and the body energy are represented in the form /3/

$$K_p = I_{pq}\omega_q = I_{pq}^\circ\omega_q + \varepsilon(K_{pqlm}' + \delta K_{pqlm}'')\omega_q\omega_l\omega_m \tag{6}$$
$$E = \frac{1}{2}I_{pq}^\circ\omega_p\omega_q + \frac{3}{4}\varepsilon(K_{pqlm}' + \delta K_{pqlm}'')\omega_p\omega_q\omega_l\omega_m$$

The components of the tensors K_{pqlm}' and K_{pqlm}'' can be calculated from the results presented in the paper. The components of the tensor K_{pqlm}'' are not presented here because of their awkwardness, while the following relationship holds for K_{pqlm}'

$$K_{pqlm}'\omega_p\omega_q\omega_l\omega_m = \frac{4}{35}\left[\frac{1}{3} - \frac{\xi_1}{3} + 2\xi_2 + 14\xi_3\right]\omega^4$$

As in the case of a solid body or an elastic quasisphere /3/, the integrals of the Euler equation $E = \text{const}$ and $\mathbf{K}^2 = \text{const}$ enable us to make a deduction about the nature of the motion of the body by means of the hodograph of the vector relative to the body, which is determined by the intersection of a centrally symmetric fourth-order surface $E = \text{const}$ with spherical surfaces ${}^4/_3E - {}^1/_2\mathbf{K}^2/I_0 = \text{const}$ corresponding to different $\mathbf{K}^2$.

Let us illustrate the results by the following two examples.

1°. A sphere in whose elastic properties there is an isotropy plane. All the directions in planes normal to the Ox_3 axis are equivalent with respect to the elastic properties and the body is isotropic in these planes. In the general case the anisotropic properties of the body are characterized by the following parameters /6/

$$a_{1133} = a_{2233} = a, \quad a_{1313} = a_{2323} = b, \quad a_{3333} = c$$

The "non-spherical" part of the expression $K''_{pqlm}\omega_p\omega_q\omega_l\omega_m$, that has the form

$$\omega_3^4 [aQ_1 + bQ_2 + cQ_3] + \omega_3^2 (\omega_1^2 + \omega_2^2) [a (Q_1 - {}^1/_2\varepsilon_{25}) + b (Q_2 - \varepsilon_{25}) + c (Q_3 + {}^1/_4\varepsilon_{25})]$$

$$Q_1 = \varepsilon_4 + \varepsilon_5 - 2\varepsilon_{15} - \varepsilon_{17}, \quad Q_2 = \varepsilon_9 + \varepsilon_{10} - \varepsilon_{19}$$

$$Q_3 = \varepsilon_1 + \varepsilon_2 - \varepsilon_{14} - \varepsilon_{15}$$

is of interest.

It can be seen that for any value of the parameter $\varkappa \in [0, \infty)$ (Poisson's ratio is $0 \leqslant \sigma < 0{,}5$) $\varepsilon_{25} > 0$, $Q_1 > 0$, while the quantities Q_2 and Q_3 are negative, hence $Q_1 - {}^1/_2\varepsilon_{25} > 0$, $Q_3 + {}^1/_4\varepsilon_{25} < 0$.

Let $a = b = 0$ and $c > 0$. The surface of the stress coefficients /6/ is an ellipsoid of revolution, the most pliable body in the Ox_1x_2 plane. We find from relationships (6) that for constant K^2 the greatest energy corresponds to rotation around Ox_3. Taking energy dissipation into account, we conclude that rotation around the axes lying in the Ox_1x_2 plane will be asymptotically stable.

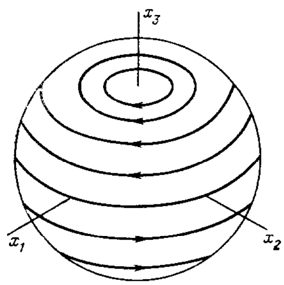

Fig.1

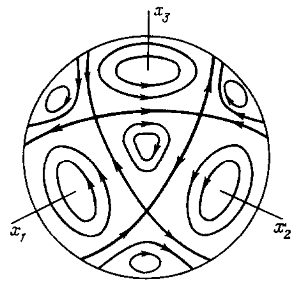

Fig.2

The hodograph of the vector ω relative to the body is shown in Fig.1.

The analysis for aribitrary values of the parameters a, b, c can be performed analogously.

2°. *A sphere whose elastic properties possess cubic symmetry.* The anisotropy is characterized by the following parameters

$$a_{1122} = a_{1133} = a_{2233} = a, \quad a_{1212} = a_{1313} = a_{2323} = b$$

$$a_{1111} = a_{2222} = a_{3333} = c$$

The "non-spherical" part of the expression $K''_{pqlm}\omega_p\omega_q\omega_l\omega_m$ has the form

$$(1 - 2\xi_1 + 6\xi_1^2)(c - a - 2b)(\omega_1^2\omega_2^2 + \omega_1^2\omega_3^2 + \omega_2^2\omega_3^2)$$

It vanishes for $c = a + 2b$, which corresponds to an isotropic elastic body.

Let $a \geqslant 0$ and (or) $b \geqslant 0, c = 0$. The surface of the stress coefficients that has the symmetry of a cube, is oriented so that the axes of the $Ox_1x_2x_3$ coordinate system pass through the centre of the faces of the "cube", i.e., the body is most pliable in the direction of these axes.

Fixing the magnitude of the kinetic moment vector, calculating its corresponding angular velocities for different stationary rotations of the sphere, we find that the energy E is minimal for ω oriented towards the middle of the faces of the cubic surface of the stress coefficients (Fig.2). As in the previous example, we conclude that only rotation around the axes of greatest pliability will be asymptotically stable.

Therefore, there is a complete analogy between the behaviour of the freely rotating anisotropic elastic sphere considered in this paper and the homogeneous isotropic elastic quasisphere studied in /3/.

The study was carried out assuming small anisotropy of the elastic properties of the body, resulting in a considerable simplification of the problem and enabling the corrections to the inertial tensor components to be evaluated. If this assumption is dropped ($\delta \sim 1$), the problem is not solvable for anisotropy of the elastic properties of general form because of difficulties of a calculational nature, although these difficulties can be overcome in a number of the simplest cases.

In conclusion, without assuming the anisotropy to be small, we again examine a freely rotating sphere in whose elastic properties there is a plane of isotropy. A quasistatic problem of the form (2) is solved

$$\partial \sigma_{ij} / \partial x_j = f_i \text{ in } V; \quad \sigma_{ij} n_j = 0 \text{ on } S$$

Here $\sigma_{ij} = \lambda_{ijkl} u_{kl}$. The following components of the tensor λ_{ijkl} are different from zero

$$\lambda_{1111} = \lambda_{2222} = \varkappa + 2, \quad \lambda_{1122} = \varkappa, \quad \lambda_{1212} = 1$$
$$\lambda_{1133} = \lambda_{2233} = a, \quad \lambda_{1313} + \lambda_{2323} = b, \quad \lambda_{3333} = c$$

The displacement vector is sought in the form of the polynomial

$$\mathbf{u} = \mathbf{q}_n x_n + \mathbf{r}_{kmp} x_1^k x_2^m x_3^p$$
$$n, k, m, p = 1, 2, 3; \; k + m + p = 3$$

Calculation of the quantities in this expression yields

$$q_{13} = q_{23}, \quad q_{11} = q_{22}, \quad q_{31} = q_{32}$$
$$r_{1300} = r_{2030}, \quad r_{1120} = r_{2210}, \quad r_{1102} = r_{2012}$$
$$r_{1030} = r_{2300}, \quad r_{1012} = r_{2102}, \quad r_{1210} = r_{2120}$$
$$r_{3201} = r_{3021}, \quad r_{3102} = r_{3012}, \quad r_{1201} = r_{2021}$$
$$r_{1021} = r_{2201}, \quad r_{1003} = r_{2003}, \quad r_{3300} = r_{3030}$$
$$r_{3120} = r_{3210}, \quad r_{1111} = r_{2111}, \quad r_{1300} = r_{1120}$$
$$r_{1210} = -\frac{a}{4b(\varkappa + 1)} \left[\frac{1}{2}(1 + b)\Omega^2 + (2 + 3b) w_1\right]$$
$$r_{1012} = -\frac{b}{4}\Omega^2 - \frac{3}{2} w_1$$
$$r_{1030} = \frac{1}{12(\varkappa + 1)} \left[\frac{a(1 + b)}{2b}\Omega^2 + \left(\frac{a(2 + 3b)}{b} - 4(\varkappa + 1)\right) w_1\right]$$
$$r_{3111} = \frac{1}{b}\left[\frac{1}{2}(1 + b)\Omega^2 + (2 + 3b) w_1\right]$$
$$w_1 = q_{12} + q_{21} = -\frac{\Omega^2}{2} \frac{(\varkappa + 1)[(a + 4b)(1 + b) - 2b^3] - a^2(1 + b)}{(\varkappa + 1)(2a + 8b + 3ab + 6b^2) - a^2(2 + 3b)}$$
$$r_{1021} = \frac{1}{2(3 + b)} [(b - 3) r_{1111} - 2w_2]$$
$$r_{1201} = \frac{a^2 - \varkappa c}{2[(\varkappa + 2)c - a^2]} r_{1111}$$
$$r_{3120} = -\frac{1}{2b(3 + b)} [2w_2 + b(b + 1) r_{1111}]$$
$$r_{3102} = \frac{a}{a^2 - (\varkappa + 2)c} r_{1111}$$
$$r_{1003} = -\frac{w_2}{3b} + \frac{a}{3[(\varkappa + 2)c - a^2]} r_{1111}$$
$$r_{3300} = -\frac{w_2}{3b} + \frac{a^2 - \varkappa c}{6[a^2 - (\varkappa + 2)c]} r_{1111}$$
$$r_{1111} = \frac{\sqrt{2}(3 + b)}{4b}\omega_3\Omega + \frac{4 + b}{b} w_2$$
$$w_2 = b(q_{31} + q_{13}) = \frac{\sqrt{2}(3 + b)}{4b}\omega_3\Omega \left[\frac{(\varkappa + 2)c - a^2}{(2b + 3)(\varkappa c - a^2) + 3c(b + 1)} - \right.$$
$$\left. \frac{4 + b}{b}\right]^{-1}$$
$$r_{1102} =$$
$$-\frac{1}{2a(a + 2b)} \left[\frac{a}{2}\Omega^2 - b\omega_3^2 - 8(\varkappa + 2) b r_{1300} + 3(a + b) c r_{3003}\right]$$
$$r_{3201} = -\frac{1}{2a(a + 2b)} \times$$
$$\left[\frac{a}{2}\Omega^2 + (a + b)\omega_3^2 + 8(\varkappa + 2)(a + b) r_{1300} - 3bc r_{3003}\right]$$
$$q_{11} = \frac{1}{4(a + 2b)[(\varkappa + 1)c - a^2]} \left\{\frac{a}{2}(c - 2a)\Omega^2 + \right.$$
$$[c(a + b) + 2ab]\omega_3^2 + 4[2b(\varkappa + 2)(2a - c) + c(a + 2b)] \times$$
$$\left. r_{1300} + 3bc(2a - c) r_{3003}\right\}$$
$$q_{33} = \frac{1}{2(a + 2b)[(\varkappa + 1)c - a^2]} \left\{\frac{a}{2}[2(\varkappa + 1) - a]\Omega^2 - \right.$$
$$[2b(\varkappa + 1) + a(a + b)]\omega_3^2 + 4[2ab(\varkappa + 2) - a(a + 2b) -$$
$$\left. 4b(\varkappa + 1)(\varkappa + 2)] r_{1300} + 3bc[a - 2(\varkappa + 1)] r_{3003}\right\}$$

$$r_{1300} = \frac{3}{2w_3}\left\{\frac{\Omega^2}{2}[2a^2(a-2b) - c(a(a+b) + 2(a-b)(\varkappa+1)) + c^2(\varkappa+1+b)] + \omega_3^2[-4a^2b + c(a(b-a) + 4b(\varkappa+1)) + c^2(\varkappa+1+b)]\right\}$$

$$r_{3003} = \frac{1}{w_3}\left\{\frac{\Omega^2}{2}[8(\varkappa+1)(\varkappa+2)b + 4a(\varkappa+2)(b-a) + 2a(2b+a-4ab+a^2) + c(a+4(\varkappa+1)(\varkappa+2) + 2(\varkappa+2)(3b-a))] + \omega_3^2[2a^2(a-b) - 4ab(\varkappa+2) - c(a+b)(2\varkappa+3)]\right\}$$

$$w_3 = 3\{-4a^2[a(a+2b) - 4b(\varkappa+2)] + 2c[2a(a+b)(2\varkappa+3) - 8b(\varkappa+1)(\varkappa+2) + ab(3a-2)] + c^2[b - 6b(\varkappa+2) - 4(\varkappa+1)(\varkappa+2)]\}$$

It is assumed here that $\omega_1 = \omega_2 = {}^1/_2\sqrt{2}\Omega$. The quantities $q_{12}, q_{21}, q_{13}, q_{31}$ can be evaluated by jointly considering the relationships $w_1 = q_{12} + q_{21}$, $w_2 = q_{13} + q_{31}$, and the condition of no motion of the body as a whole $\int_V \mathbf{r} \times \mathbf{u}\,dV = 0$ [3]. in the system $Ox_1x_2x_3$.

The expressions presented for q_{ij}, r_{ijkl} enable us to find the corrections to the inertia tensor due to the anisotropies of the elastic properties of the sphere from the formulas

$$I_{11}' = I_{22}' = \frac{2}{35}\{7(q_{11} + q_{33}) + 4r_{1300} + 3r_{3003} + r_{1102} + 2r_{3201}\}$$

$$I_{33}' = \frac{2}{35}\{7q_{11} + 4r_{1300} + r_{1102}\}$$

$$I_{12}' = -\frac{1}{35}\{7w_1 + 6r_{1030} + 2r_{1210} + 2r_{1012}\}$$

$$I_{13}' = I_{23}' = -\frac{1}{35}\{7w_2 + 3r_{3300} + 3r_{1003} + r_{3120} + r_{1021} + r_{3102} + r_{1201}\}$$

An investigation of the small anisotropy case yields a representation of the qualitative dynamics of the sphere.

The author is grateful to G.G. Denisov for his interest.

REFERENCES

1. CHERNOUS'KO F.L., Influence of intrinsic elasticity and dissipation on the motion of a solid relative to the centre of mass, Dynamics of Continuous Media, 41, 118-122, Izd. Inst. Gidrodinamiki Sibir. Otdel Akad. Nauk SSSR, Novosibirks, 1979.
2. EGARMIN N.E., Influence of elastic deformations on the inertia tensor of a solid, Izv. Akad. Nauk SSSR, Mekhan. Tverd. Tela, 6, 1980.
3. DENISOV G.G. and NOVIKOV V.V., On the free motions of a deformable solid, close in shape to a sphere, Izv. Akad. Nauk SSSR, Mekhan. Tverd. Tela, 3, 1983.
4. ZIGEL' F.YU., To You, Earthling, Nedra, Moscow, 1983.
5. DEMIDOV S.P., Theory of Elasticity, Vyssh. Shkola, Moscow, 1979.
6. LEKHNITSKII S.G., Theory of Elasticity of an Anisotropic Body. Gosteorizdat, Moscow, 1950.

Translated by M.D.F.